A method for determining noise coupling in a phased array antenna

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Abstract—
A method for the calculation and accurate prediction of the noise coupling in a phased antenna array is introduced. The noise coupling is shown to be related to the scan reflection coefficient, a measurable antenna parameter. This leads to a method of calculating the noise coupling of the array using scattering parameters, which can then be used in design of receive-mode phased array. A 49-element dipole array at 1 GHz using HPMGA-82563 LNAs is used as an example to calculate the parameters of interest for a low-noise phased array design.

I. Introduction

In the Westerbork radio-telescope array in the Netherlands, a two-element dipole array was used as the dish feed. When the beam was pointed to a dark (low-noise) region of the sky, it was found that instead of an decrease in input noise, the noise increased [1]. The problem is attributed to noise coupling between the receiving dipoles and LNAs. The motivation for the work presented in this paper is a very large phased array radio-telescope [2] in which the noise coupling problem may be critical.

In most phased array antennas, element spacing is on the order of a half wavelength ($\lambda_0/2$) at the operating frequency. The close spacing of the elements leads to electromagnetic coupling between them. As the main beam of the array is scanned, the coupling varies. One way of characterizing this coupling is by the scan reflection coefficient, which is a reflection coefficient at each antenna element as a function of scan angle. The scan reflection coefficient measures the ratio between the wave incident at the antenna feed and the wave scattered from the antenna element and other array elements. When each antenna element also contains a low-noise amplifier (LNA), noise is also

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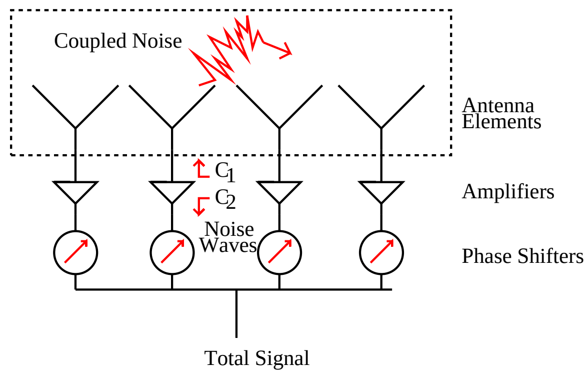

Fig. 1. Illustration of noise coupling in the receive part of a phased array antenna. The noise waves at each LNA input and output are c_1 and c_2, respectively. The incident and scattered waves, a_i and b_i, are used to define the scan reflection coefficient.

coupled from one element to the other. This can lead to very high noise levels at some scan angles.

The mechanism of noise coupling in active antenna arrays is described schematically in Fig.1. Noise is generated by an LNA both at its input and output ports . The noise that is generated at the input port of the LNA radiates and couples to the other antenna elements, contributing to their total received noise power. The noise coupling coefficients, $N_c(\theta, \phi)$, are a function of the scan angle, since the relative phase between elements changes as the array is scanned. In this paper, we show that the noise coupling is proportional to the scan reflection coefficient $\Gamma(\theta, \phi)$ of the array. This conclusion is reached using both a physical and mathematical argument. The usefulness of the conclusion lies in the fact that $\Gamma(\theta, \phi)$ is an easily measurable quantity, while no method currently exists for noise coupling measurement in large arrays.

II. Scan Reflection Coefficient of an Array

The scan reflection coefficient of a phased array describes the effect that scanning has on the input match of each array element. It is defined for both finite and infinite arrays. For finite arrays, is is generally defined for the center element [3].

Consider a wave transmitted by an N-element array in a direction (θ_o, ϕ_o), which requires that the input waves at the antenna feeds are give by:

$$\mathbf{a}(\theta_\mathbf{o}, \phi_\mathbf{o}) = \left[\begin{array}{cccc} e^{j\beta_1} & e^{j\beta_2} & \cdots & e^{j\beta_N} \end{array} \right]^\mathbf{T}, \quad (1)$$

where the superscript T denotes the transpose operation. (In this paper, capital letters will refer to matrices, bold lower case letters to vectors, and plain lower case to scalars.) The reflected waves at all of the antenna ports (elements) are $\mathbf{b}(\theta_o, \phi_o) = S\mathbf{a}(\theta_o, \phi_o)$, where S is the scattering matrix of the array, as indicated with a dashed line box in Fig.1. The parameters of this $N \times N$ matrix can be measured using a network analyzer, or can be found using any commercial antenna simulation tool. The vector $\mathbf{b}$ is a function of θ_o and ϕ_o, the scan angle of the array. The active reflection coefficient of the i-th array element is defined as:

$$\Gamma_i(\theta_o, \phi_o) = \frac{b_i(\theta_o, \phi_o)}{a_i(\theta_o, \phi_o)} \quad (2)$$

The value $b_i(\theta_o, \phi_o)$ is equal to the inner product of the ith row of the matrix S, denoted by $S(i, :)$, with the vector $\mathbf{a}(\theta_\mathbf{o}, \phi_\mathbf{o})$. This gives the short hand notation of the active reflection coefficient as:

$$|\Gamma_i(\theta_o, \phi_o)| = |< S(i, :), \mathbf{a}(\theta_\mathbf{o}, \phi_\mathbf{o}) >| \quad (3)$$

The array performs well under scanning when the active scan reflection coefficient has a small amplitude, i.e. when the near-field coupling between elements is low for all scan angles.

III. Noise in a Gain-Matched Array

When LNAs are used in a phased array, they add noise to the signal. A useful way of characterizing noise in a microwave circuit is to extend the notation for scattering parameters to also include noise. At a given frequency f_o and a small bandwidth Δf a noise wave can be written as a complex number c. The LNA can then be described by its 2-port scattering matrix as [4]:

$$\mathbf{b} = \mathbf{S}\mathbf{a} + \mathbf{c}, \quad (4)$$

where the $\mathbf{c}$ represents the noise waves generated at the LNA ports. The magnitude of the noise wave is the square root of the time average noise power [5].

In a standard noise model of an array the radiated noise c_1 is ignored. This is the noise that is produced at the input port of the LNA. In a large phased array, this radiated noise can significantly degrade the noise performance of the array, since it is coupled to the other antenna elements and amplified. The coupled noise also undergoes a phase shift at the beam former, which is a function of the scan angle of the main beam.

One method of analyzing noise coupling in an array is to use one noisy amplifier in the analysis of the array, and $N-1$ noise-free LNAs (0 K). For a finite array, the noisy element is placed in the center of the array. The total noise figure of the array is found as follows. A noise wave is inserted into the array at the ith element, and its coupling to the other elements is found from the scattering matrix as:

$$\mathbf{C_c} = \mathbf{S} \left[\begin{array}{ccccccc} 0 & \cdots & 0 & c_1 & 0 & \cdots & 0 \end{array} \right]^\mathbf{T}. \quad (5)$$

The vector $\mathbf{C_c}$ represents the noise waves in each array element feed between each antenna and LNA. Assuming that all LNAs have equal voltage gains, G, the coupled noise is equal to $Gc_1S(i, :)$ at the output of each element LNA and at the input of the phase shifters. For a main beam in the direction (θ_o, ϕ_o) a phase β_k is added to all waves incident on the kth element. Note that the phase shifts added to the noise waves are the same as those of the input wave $\mathbf{a}$ used for the calculation of the active reflection coefficient. The coupled noise wave at the output of the phase shifters of each element is now:

$$\mathbf{n_c}(\theta_o, \phi_o) = \mathbf{n_2} + Gc_1 S_{i,k} e^{j\beta_k} \quad (6)$$

The scattered waves at all the ports are then combined into a single signal, n_{tot}, which is written as:

$$\begin{aligned} n_{tot}(\theta_o, \phi_o) &= c_2 + \sum_{k=1}^{N} Gc_1 S_{i,k} e^{j\beta_k} \quad (7) \\ &= c_2 + Gc_1 < S(i, :), \mathbf{a}(\theta_\mathbf{o}, \phi_\mathbf{o}) > (8) \end{aligned}$$

Substituting Eq.(3) for the active reflection coefficient gives

$$n_{tot}(\theta_o, \phi_o) = c_2 + Gc_1 e^{-j\beta} \Gamma_i(\theta_o, \phi_o) \quad (9)$$

The total noise coupling as a function of the scan angle is directly proportional to the active reflection coefficient. The proportionality constant is given by c_1 and G, which are properties of the specific LNA and can be calculated from the specification sheets provided by the manufacturer ($\Gamma_{opt}, R_n, NF_{min}$). We define the noise coupling ratio as the ratio of the coupled noise $Gc_1\Gamma_i$ to the standard noise c_2.

IV. Noise in a Noise-Matched Array

In the previous section, the amplifiers were assumed to be matched at the input to the antennas, and the noise match was not taken into account. We now show that the noise coupling that was found in Eq.(9) is a first order, but good approximation to the rigorous solution. In the receive array with integrated LNAs, the noise characteristics of the array can be derived from the noise characteristics of the amplifier along with the s-parameters of the amplifier and array. To include the noise match in the active array, consider the multiport cascade in Fig.2. The N-port labeled S_A in the figure represents the antennas feeds. The feed ports are connected to N amplifier input ports. The N output ports are the outputs of the LNAs that later in the system connect to the phase shifter network. The calculation of the scattering matrix of the active array S_{net}, is performed by cascading the scattering matrix of the antenna array S_A with that of the individual amplifier arrays S_{LNA}, where

$$S_{LNA} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \tag{10}$$

The scattering matrix of the N amplifiers can be rewritten as a $2N \times 2N$ block diagonal matrix S_{Amp}, where the diagonal blocks are S_{LNA}. The fist step in calculating the noise of the cascaded system is to group the matrices of the two multiports into a single scattering matrix:

$$\left\{ \begin{array}{c} \mathbf{b}_{Amp} \\ \mathbf{b}_A \end{array} \right\} = \begin{bmatrix} S_{Amp} & 0 \\ 0 & S_A \end{bmatrix} \left\{ \begin{array}{c} \mathbf{a}_{Amp} \\ \mathbf{a}_A \end{array} \right\} \tag{11}$$

After some algebraic manipulations, which cannot be detailed in this short paper, the total noise at the N output ports is

$$\mathbf{c}_{net} = \mathbf{c}_2 + s_{21}S_A[I - s_{11}S_A]^{-1}\mathbf{c}_1 \tag{12}$$

where $\mathbf{c}_1$ and $\mathbf{c}_2$ represent N input and output amplifier noise waves, respectively. This is now a complete solution for the noise at the output ports of the active antenna array. For the case of a gain-matched amplifier, $s_{11} = 0$, Eq.(12) simplifies to $\mathbf{c}_{net} = \mathbf{c}_2 + s_{21}S_A\mathbf{c}_1$, the same value derived using the approximate approach from Section 3. However, it cannot be concluded from Eq.(12) how good the approximation is.

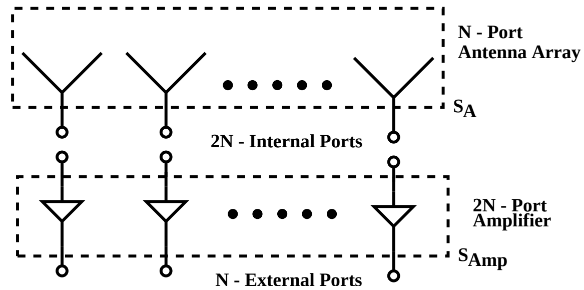

Fig. 2. Multiport representation of the connection of an N-element antenna array to N amplifiers. When evaluating the noise coupling, the internal ports are connected reducing the size of the problem to $N \times N$.

For the noise-matched LNA, the inverse of $I - s_{11}S_A$ is still present in Eq.(12). Since S_A represents a passive network, the eigen values of $s_{11}S_A$ satisfy $\lambda_i \leq |s_{11}| << 1$, and therefore

$$[I - s_{11}S_A]^{-1} = I + s_{11}S_A + s_{11}^2 S_A^2 + \cdots \tag{13}$$

which combined with Eq.(12) gives

$$\mathbf{c}_{net} = \mathbf{c}_2 + s_{21}S_A\left[I + s_{11}S_A + s_{11}^2 S_A^2 \cdots\right]\mathbf{c}_1 \tag{14}$$

Therefore, for small values of the input mismatch, Eq.(14) reduces to Eq.(6) in the first approximation, validating the simple approach of calculating noise coupling presented in the previous section.

V. Example of Noise Coupling in a 49-Element Dipole Array

As an illustration of the usefulness of the presented noise coupling theory, consider a 7×7 half-wave dipole array. The full scattering matrix of the array is calculated with a Method of Moment (MoM) integral equation solver [6]. The array consists of 49 half wavelength dipoles above a ground plane in a rectangular 7×7 grid with a $0.55\lambda_0$ period. The scan reflection coefficient is calculated as described in Sec.II.

As observed from Eq.(9), the LNA noise figure, noise resistance and optimal noise match directly affect the noise coupling coefficients. For this example, we assume that a Hewlett Packard MGA-82563 low-noise amplifier is connected to each of the 49 dipoles. The noise parameters for this LNA at 1.0 GHz are, $NF_o = 2.10$dB, $\Gamma_{opt} = 0.15\angle 45^o$ and $R_N = 6.0$, and the gain is 14. dB. Using formulas given in [5], magnitudes of the noise waves are found to be $|c_1| =$

$5.4023 \cdot 10^{-11}\,\mathrm{V}/\sqrt{Z_o}$ and $|c_2| = 3.2690 \cdot 10^{-10}\,\mathrm{V}/\sqrt{Z_o}$, and the ratio $|c_1/c_2| = 0.1653$.

An interesting figure of merit is the amount of coupling that produces 50% more noise power at the array output at certain scan angles than expected without noise coupling. To find the minimum amount of coupling that would give the noise ratio of -3dB, we set

$$10 \log \frac{|\Gamma(\theta,\phi) G c_1|^2}{|c_2|^2} = -3\,\mathrm{dB} \tag{15}$$

Using Eq.(8) we can solve for the minimum scan reflection coefficient that would give a noise coupling ratio of greater then -3 dB. The voltage gain of the LNA is 5.0, the ratio of the noise waves is 0.17. we then solve

$$\Gamma G c_1 / c_2 = -3\mathrm{dB} \tag{16}$$

which yields $\Gamma_i = 0.86$ or -1.4 dB. This is not very hard to achieve practically, however gain larger than 14 dB is usually of interest. An LNA with a higher gain would have a significantly lower threshold of acceptable Γ_i that would yield a coupling ratio of less then 3 dB. A cross-section of the matrix S_A is shown in Fig.3 for one row of the array, showing the level of coupling between the elements as calculated by the MoM. Fig.4 shows the scan reflection coefficient, and therefore the noise coupling of the center element of the array. The light areas in the plot show high coupling for the corresponding scan angle, indicating that the practical scanning is limited, in the E-plane to a relatively small field of view.

The scan reflection was also calculated for a 7×7 dipole array with an array spacing of one λ_0. In this case, as expected, the noise coupling is much smaller, and for low noise coupling , one should choose an array with larger element spacing. Unfortunately, this will limit the scan angle due to grating lobes. For example, for a beam scanned in the E-plane to 45^o, the half-wavelength spacing array has no grating lobes and a noise coupling ratio of 0.64. However, an array with full wavelength spacing with a noise coupling ratio of 0.3 will have a grating lobe at -15^o when the main beam is scanned to 45^o. Therefore, there is a tradeoff in designing low-noise scanning arrays: when the noise coupling is low, the scan angle is limited, and when the scan angle is wide, there can be significant noise coupling at the large scan angles. Design parameters that can be adjusted for minimizing the noise coupling are the antenna element radiation pattern, LNA properties, and array lattice.

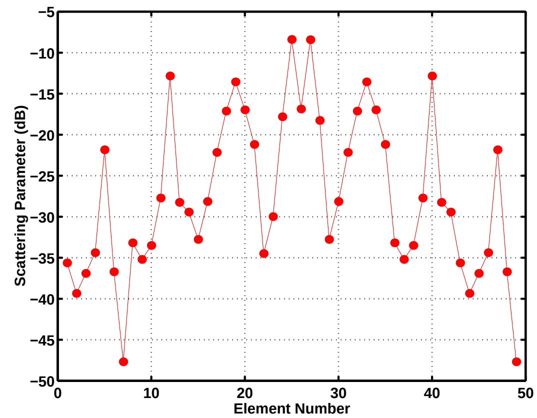

Fig. 3. Magnitude of elements in a single row of the scattering matrix S_A as calculated by a MoM code. The relative magnitudes indicate levels of near-field coupling between elements.

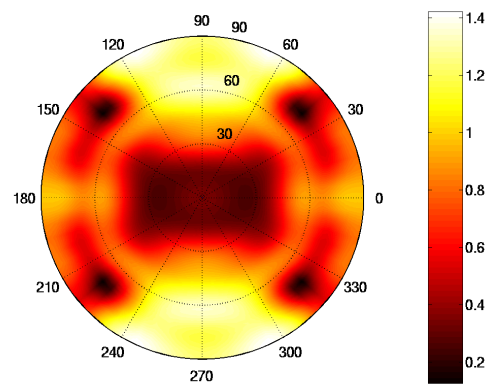

Fig. 4. Plot of the magnitude of the scan reflection coefficient for the center element of a 7×7 dipole array with a $\lambda_0/2$ spacing. The polar plot represents the scan angles as azimuth and elevation. The dipole is oriented in the 180^o-0^o direction.

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